Mathematics of Finance

B.Com (P) – Year 1, Sections B & C

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Mathematics of Finance

- Topics covered so far in class:

1. Simple Interest
2. Compound Interest
3. Nominal and Effective Rates of Interest
4. Discount
5. Present Value
6. Equation of Value

- Topics left:

1. Depreciation

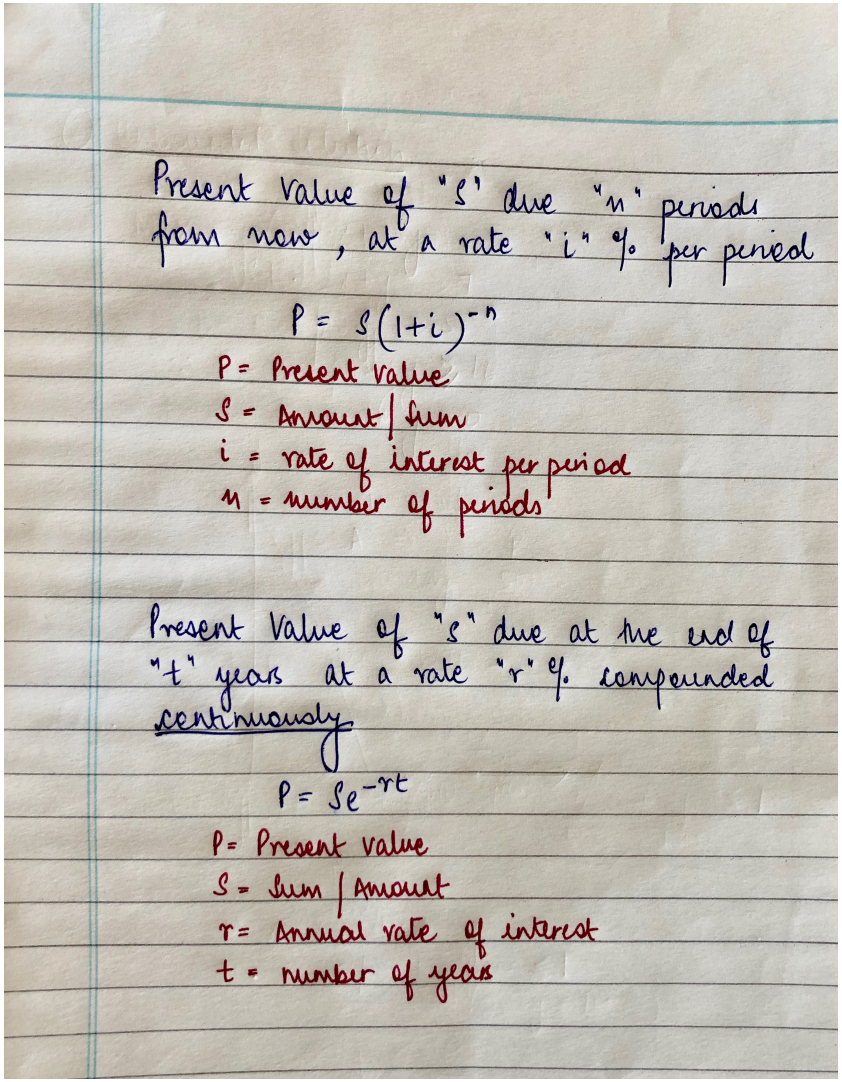
Let's Revise..

- Present Value

- While planning for the future, we may want to know *how much to invest now* to be able to save an amount "S" at a later date.
- Therefore, we want to find out the *principal* originally invested.

- Eg: I want to save Rs.6,00,000 for my education, which I will need after 5 years. How much should I put in the bank today, if the interest is compounded at 6% annually?
- *The amount that I will put in the bank today is called Present Value*

Present Value Formula



Present value of "S" due "n" periods from now, at a rate "i" % per period

$$P = S(1+i)^{-n}$$

P = Present value
S = Amount / Sum
i = rate of interest per period
n = number of periods

Present Value of "S" due at the end of "t" years at a rate "r" % compounded continuously

$$P = Se^{-rt}$$

P = Present value
S = Sum / Amount
r = Annual rate of interest
t = number of years

Equation of Value

- We obtain this by equating the sum of values on one date with another set of values on another date.
- The date on which these values are equal is called the *Focal Date* or *Comparison Date*.
- Focal Date can be decided by the lender or the borrower

For Example…

- Mr. X has an outstanding loan of Rs.10,000 which is due 5 years from now.
- He decides to repay this loan in installments by paying Rs.2,000 today, Rs.4000 after 3 years and the balance after 6 years @ 6% per annum.
- If the lender decides the focal date to be 6 years from now.
- The equation of value will mean that *the value of the old obligation of Rs.10,000 on the focal date (i.e. 6 years from now) = value of Rs. 2,000 on focal date + value of Rs.4,000 on focal date + value of remaining amount on focal date.*

- Solution on next page
- Also, refer to video sent on class group for full explanation

EQUATION OF VALUE
(EXAMPLE FROM PPT)

Let focal date = 6 yrs from now
Let balance paid = ₹ x

$r = 6\%$ p.a.

Old Obligation	val. of old eb. on F.D.	New obligat^{n}	val of new eb. on F.D.
₹10,000 @ end of 5 yrs.	$10,000(1+0.06)^1$	₹2000 due today	$2000(1+0.06)^6$
		₹4000 @ end of 3 yrs.	$4000(1+0.06)^3$
		₹x @ end of 6 yrs	x.

$10,000(1+0.06)^1 = 2000(1+0.06)^6 + 4000(1+0.06)^3 + x$ } $x = 2999$

$10,000(1.06) = 2000(1.06)^6 + 4000(1.06)^3 + x$

$10,600 = 2000(1.4185) + 4000(1.1910) + x$

$\therefore x = ₹2999$

Q. A debt of Rs.2000 due in 3 years and Rs.3000 due in 7 years is to repaid by a single payment of Rs.1000 now and two equal payments which are due 1 year from now and 4 years from now. If the interest rate is 6% compounded annually, how much should each of the payments be?

- Let the value of each payment be Rs.x
- Let the Focal Date be 4 years from now
- Rate of interest is 6% p.a.

Old Obligation	Value of Old Obligation on Focal date	New Obligation	Value of New Obligation on Focal Date
Rs. 2000 after 3 years	$2000(1+0.06)^{1}$	Rs. 1000 today	$1000(1+0.06)^{4}$
Rs. 3000 after 7 years	$3000(1+0.06)^{-3}$	Rs.x after 1 year	$X(1+0.06)^{3}$
	This is $3000(1+0.06)^{-3}$ because the debt is due 7 years from now but focal date is 4 years from now. We will go back 3 years to reach focal date.	Rs.x after 4 years	X

- Equation of Value will be given by:
- $2000(1+0.06)^{1} + 3000(1+0.06)^{-3} = 1000(1+0.06)^{4} + X(1+0.06)^{3} + X$
- X = Rs. 1541.02 (value of each installment)

Depreciation

- Refers to the annual wear and tear of machinery, equipment, etc, due to use and passage of time.
- It is actually a decrease in the value of an asset due to the above mentioned reasons.

- Depreciation can be calculated by the methods given below:

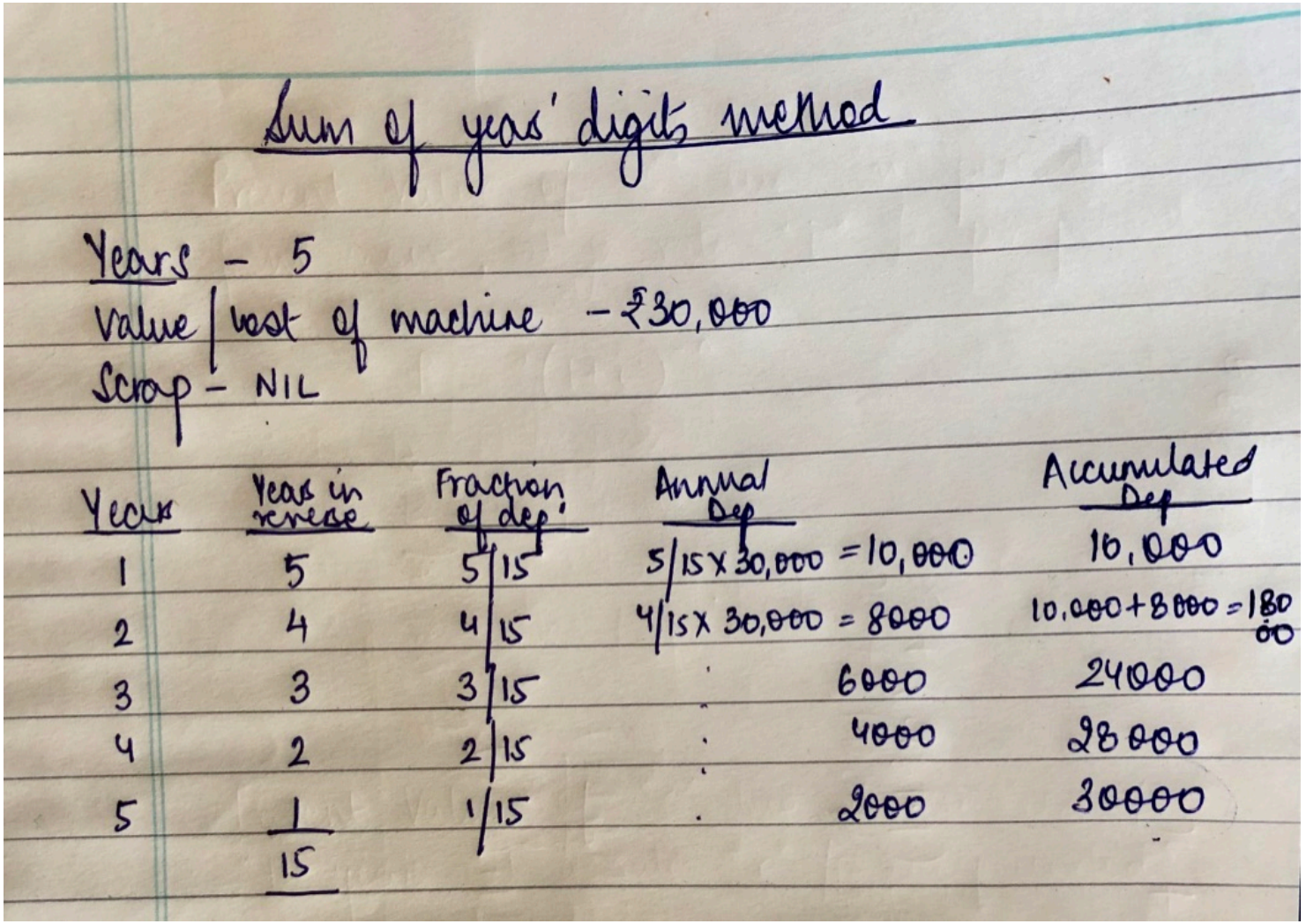
a. Sum of the Years' Digits Method
b. Straight Line Method
c. Diminishing Balance Method

Sum of the Years' Digits Method

- An asset costing Rs.30,000 is expected to have no scrap value and a useful life of 5 years. Find annual depreciation using sum of the years' digits method and prepare a depreciation schedule.

- Refer to next slide for solution
- Refer video sent on class group for full explanation

Solution…



Sum of years' digits method

Years – 5

Value / cost of machine – ₹30,000

Scrap – NIL

Year	Years in reverse	Fraction of dep.	Annual Dep	Accumulated Dep
1	5	5/15	5/15 × 30,000 = 10,000	10,000
2	4	4/15	4/15 × 30,000 = 8000	10,000 + 8000 = 18000
3	3	3/15	6000	24000
4	2	2/15	4000	28000
5	1	1/15	2000	30000
	15			

SLM – Straight Line Method
DBM – Diminishing Balance Method

DEPRECIATION:

C – Original cost
S – Scrap
W – Total dep^n
D – Annual dep
n – Useful life.

SLM: $D = \frac{C-S}{n} = \frac{W}{n}$

DBM: r ⇒ constant rate of dep^n (% form)

@ end of	Dep^n	Cr
1st yr.		Cr
2nd yr	Cr(1-r)	C(1-r) − Cr(1-r) = C(1-r)^2

BV = C − Cr = C(1-r)

Straight Line Method

- A machine costing Rs.30,000 is expected to have a useful life of 5 years and scrap value of Rs.5,000. Find annual depreciation using straight line method and prepare depreciation schedule.

Cost (C) = 30000
Scrap (S) = 5000
Useful Life (n) = 5

Depreciation (D) = C-S/n
D = (30000-5000)/5
D = 5000

Years	Annual Depreciation	Accumulated Depreciation	Book Value at end of Year
1	5000	5000	30000 – 5000 = 25000
2	5000	10000	25000 – 5000 = 20000
3	5000	15000	20000 – 5000 = 15000
4	5000	20000	15000 – 5000 = 10000
5	5000	25000	10000 – 5000 = 5000 (scrap)

Diminishing Balance Method / Reducing Balance Method / Written Down Value Method

- Depreciation is calculated on the book value of the asset
- Under this method, depreciation is a constant percentage of BV
- The asset is depreciated till the end of its estimated useful life
- In the end, we are left with scrap value, if any

- Refer next slide for formula
- Refer video sent on class group for explanation

Dep^{n} on WDV

C = cost of asset

r = annual rate of dep^{n}

	Cost/B.V.	Dep^{n}	B.V. @ end of the yr
1st yr.	C	C.r	$C - Cr$ $= C(1-r)$
2nd yr.	$C(1-r)$	$C(1-r) \cdot r$	$C(1-r) - C(1-r) \cdot r$ $= C(1-r)(1-r)$ $= C(1-r)^2$
3rd yr			$C(1-r)^3$

$\therefore \quad n^{th} = C(1-r)^n$

$S = C(1-r)^n$

A machine costing Rs.70000 depreciates at a constant rate of 6%. What is the depreciation charge for the 9th year?

If the estimated life of the machine is 10 years, determine the scrap value of the machine

Cost of Machine (C) = ₹ 70,000
Rate of Depreciation (r) = 6% = 0.06
Estimated life of machine = 10 years

(i) Depreciation charge for 9th year
= Dep^n Value of machine @ end of 8th Year
− Value of machine @ end of 9th Year

We know that $S = C(1-r)^n$

Val. of mach @ end of 9th Year = $70000(1-0.06)^9$
= 70,000 (0.5729)
= 40,103

Val. of mach @ end of 8th Year = $70000(1-0.06)^8$
= 70000 (0.6095)
= 42,665

∴ Dep^n charge for 9th Year = 42,665 − 40,103
= 2562

Book Val. of mach. @ end of 8th Year
− Book val. of mach. @ end of 9th Year

(ii) Scrap Value of Machine

$S = C(1-r)^n$
C = 70000
r = 0.06
n = 10

$S = 70,000(1-0.06)^{10}$
= 70,000 (0.5386)
= 37,702

A machine depreciates at 10% per annum for the first two years and then at 7% per annum for the next 3 years on diminishing balance. If the initial value of the machine is Rs.10000, find the depreciated value of the machine at the end of the 5th year.
Also, find the average rate of depreciation

- Depreciated Value of the machine at the end of the 5th year

$S = C (1 - r)^n$

$= 10000 (1 - 0.10)^2 (1 - 0.07)^3$

$= 10000 (0.90)^2 (0.93)^3$

$= 10000 (0.81) (0.804357)$

= Rs. 6515.29

- Average rate of Depreciation

Let the average rate of depreciation be "r" %

Scrap Value (as calculated above) = Rs. 6514.83

$S = C (1 - r)^n$

$6515.29 = 10000 (1 - r)^5$

$6515.29/10000 = (1 - r)^5$

Taking log on both sides

$\text{Log } (6515.29/10000) = \text{Log } [(1 - r)^5]$

Using properties of Log

$\text{Log } (6515.29) - \text{Log } (10000) = 5 \text{ Log } (1 - r)$

Using Log Tables as taught in Class

$3.4624 - 4.0 = 5 \text{ Log } (1 - r)$

$-0.5376 = 5 \text{ Log } (1 - r)$

$-0.10752 = \text{Log } (1 - r)$

Using Antilog Tables as taught in class

$\text{AL } (-0.10752) = \text{AL } (\text{Log } (1 - r))$

$0.9177 = 1 - r$

$r = 0.0823$

$= 8.23\%$

- Kindly practice **all** unsolved questions given in the book, **especially** those which have been asked previously in exams.
- For any questions or doubts, I will be available on call/message on Thursday during your respective class timings.
- Next week's notes on a new chapter will be uploaded soon.